Stress Errors Associated with Miniaturization of X-Ray Stress Analyzer

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ABSTRACT

The problems caused by miniaturization of the goniometer are discussed from the viewpoint of the stress error caused by mis-setting of the specimen and misalignment of the collimator and detector for the Ω assembly X-ray stress analyzer using a position sensitive detector. The stress errors were calculated using a model and a simulation method. It was found that they tend to increase with the decrease in goniometer radius and effective collimator length. Therefore, the minimum size of the analyzer was investigated for a given value of total error. In conclusion, it is believed that the size for the goniometer radius and the collimator effective length could possibly be optimized. An analyzer of the single exposure method was also studied and found sensitive to the misalignment of the collimator and detector, but not to specimen mis-setting for miniaturization.

INTRODUCTION

There are many mechanical parts whose reliability depends heavily on their residual stresses. It is also true that most of these parts cannot be measured for residual stress because of size limitations. Therefore, the miniaturization of an X-ray stress analyzer is one important subject in this field. Although various efforts have been made, the current minimum size of an analyzer seems to be determined mainly by the miniaturization of the X-ray tube and the detector. Little discussion has been held on the size of the X-ray path from X-ray focus to detector through the collimator and specimen.

In this paper, the problems caused by miniaturization of the goniometer are discussed from the viewpoint of the stress error caused by mis-setting of the specimen and misalignment of the collimator and detector for the Ω assembly X-ray stress analyzer using a position sensitive detector (PSD). The stress errors are calculated using a model and a simulation method [1]. The single exposure method is also investigated.

MODELING AND SIMULATION OF X-RAY STRESS MEASUREMENT

X-ray Stress Analyzer

The strain measurement in the ψ direction using an X-ray stress analyzer of Ω assembly is schematically illustrated in Fig.1. The PSD is fixed at the diffraction angle 2θ₀ of the specimen with zero stress as its center and normal to the diffraction plane.
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The goniometer radius $R_0$ is defined as the distance from the goniometer center O to the PSD. The incident angle of X-rays is $\psi_0$ and $\psi_0 = \psi - \eta$, where $\eta = \pi / 2 - \theta_0$. The amount of specimen mis-setting is defined as the offset $L$ as shown in Fig.1. The sign of $L$ is same as that of the $z$-axis.

**Modeling**

Figure 2 shows the incident X-ray beam of $\psi_0$ through the collimator to the specimen. Let us consider an X-ray beam UD and define it by the distance $x'_{\psi}$ from the center of the incident beam at the entrance of the collimator and the distance $x_{\psi}$ from the center of the incident beam at the specimen surface $O'$. The sign of $x'_{\psi}$ and $x_{\psi}$ is same as that of the $x$-axis. Point $O''$ is the point hit by the X-ray. So, the increase in the incident angle $\Delta \psi$ of the X-ray is expressed as

$$\Delta \psi = \tan^{-1} \frac{x'_{\psi} - x_{\psi}}{CI - L / \cos \psi_0},$$

where $CI$ is the effective length of the collimator and is given as the distance from the entrance to the goniometer center O. The X-ray gives the diffraction angle of $2\theta'$ to a specimen with stress $\sigma_1$ and principal stresses of $\sigma_1$ and $\sigma_2$, which is written as follows:

$$2\theta' = \frac{\sigma}{K} \sin^2(\psi + \Delta \psi) + 2\theta_0 - \frac{\nu}{K(1 + \nu)} \cdot (\sigma_1 + \sigma_2),$$

where $K$ is the X-ray stress constant and $\nu$ is Poisson’s ratio.

Figure 3 illustrates the path of X-ray UD to the PSD. $\angle BOO'$ is $2\theta_0$ and the position B corresponds to the diffraction angle of $2\theta_0$ on PSD. Let us consider the position C of $2\theta$ on PSD and show $\angle BOC$ by $\beta_0(2\theta)$ and $\angle B''O''C$ by $\beta(2\theta)$. $B''O''$ is parallel to BO. The diffraction intensity at point C given by the X-ray can be written as follows by a Gaussian distribution of $\beta(2\theta)$ at $\beta(2\theta_0)$ as its center:

$$I_{2\theta} = I_0 \cdot \sec^2 \beta_0(2\theta) \cdot \cos \beta(2\theta) \cdot \left[ \frac{R_0 \cdot \cos \beta(2\theta)}{R} \right]^2 \cdot \exp \left[ -b^2 (\beta(2\theta) - \beta(2\theta_0))^2 \right],$$

where $I_0$ is the diffraction intensity at point B given by the X-ray UD and $R$ is the distance between the point $O''$ irradiated by the X-ray to the point C on PSD. $b$ is the parameter controlling the half-value breadth ($HVB$) of X-ray diffraction profile of specimen. $2\theta_0$ is the diffraction
angle of the specimen under mis-setting and expressed as

\[ 2\theta_1 = 2\theta_0 + \tan^{-1} \left( \frac{x_{\psi} \cdot \cos \Delta \psi}{\cos(\psi_0 + \Delta \psi)} \cdot \sin(2\theta_0 - \pi / 2 - \psi_0) - R \cdot \tan \Delta \psi + R \cdot \tan(2\theta_1 - 2\theta_0) \right) \]

The diffraction intensity of the X-ray beam of the incident angle \( \psi_0 \) at \( 2\theta \) is given by integrating \( I_{2\theta} \) at \( x_{\psi} \) and \( x_{\psi} \) from \( -b_{\psi} \) to \( b_{\psi} \) for both \( x \). As shown in Fig.4, if the collimator is settled with a misalignment of translation \( t \) and/or rotation \( \kappa \), the diffraction profile is given by the integration method corresponding to each misalignment [1].

**Conditions for Simulation**

It was assumed for the simulation that ferrite steels are measured using a chromium target with four incident angles of 0, 15, 30 and 45 deg. The values used of \( 2\theta_0 \) and \( K \) are 156.4 deg and -318 MPa/deg, respectively.

**MINIATURIZATION**

**Reduction of the Goniometer Radius**

It was found that the absolute value of the X-ray stress for the zero stress specimen increases with an increase in width of the collimator and/or a decrease in the effective length. Let us call such apparent stress the intrinsic stress \( \sigma_c \) of the collimator [1].

For a typical collimator of \( 2b_{\psi} = 1 \) mm and \( Cl = 135 \) mm, and a specimen of \( HVB = 2 \) deg, the change in the intrinsic stress was calculated with the change in the goniometer radius. Figure 5 shows the plotting of the intrinsic stress against the goniometer radius. It is seen that the value of the intrinsic stress rapidly increases when the radius becomes less than 30 mm. Therefore, let us consider the limit of the goniometer radius to be \( R_0 = 30 \) mm.

It was found that the error in the X-ray stress \( \Delta \sigma_L \) for the offset \( L \) due to specimen mis-setting can be expressed by the following equation [1]:

\[ \Delta \sigma_L = -4.59 \times 10^3 \left( 1 - 0.41 \left( \frac{2b_{\psi}}{Cl} \right) - 8.39 \left( \frac{2b_{\psi}}{Cl} \right)^2 \right) \cdot \frac{L}{R_0} \text{ (MPa).} \]  

Using Eq.(5), the stress error \( \Delta \sigma_L \) was calculated and found that an error of about 15 MPa occurs.
for \( L = 0.1 \) mm. The error in the X-ray stress \( \Delta \sigma_r \) due to the translation \( t \) of the collimator can be expressed by the following equation [1]:

\[
\Delta \sigma_r = -1.1 \times 10^4 \frac{t}{R_0} \text{ (MPa).} \tag{6}
\]

Using Eq.(6), the stress error \( \Delta \sigma_r \) was calculated and plotted against \( t \) in Fig.6. It is seen in the figure that an error of about 20 MPa occurs for \( t = 0.05 \) mm. The stress error \( \Delta \sigma_\kappa \) due to the rotation \( \kappa \) of the collimator can be expressed by the following equation [1]:

\[
\Delta \sigma_\kappa = 5.71 \times 10^3 \left( 1 + 2.89 \times 10^{-4} \cdot C \left( C \right) + 3.57 \times 10^{-4} \cdot C^2 \right) \frac{\kappa}{R_0} \text{ (MPa).} \tag{7}
\]

Using Eq.(7), the error \( \Delta \sigma_\kappa \) was calculated and it was found that an error of about 20 MPa occurs for \( \kappa = 0.025 \) deg.

The misalignment of the PSD was further investigated for cases of translation in the directions of goniometer radius and diffraction angle, and the rotation at its center. It was found that little error occurs even for error on the order of 1 mm and 1 deg of translation and rotation, respectively.

**Reduction of Collimator Effective Length**

Next, the effect of the reduction of collimator effective length to 30 mm was investigated. The intrinsic stress was calculated for collimators of various widths. Figure 7 shows the intrinsic stress and half-value breadth \( \text{HVB} \) of the profile at \( \psi_0 = 0 \) deg plotted against \( 2b_\psi \). It is seen that a collimator of 1 mm width has an intrinsic stress of -5 MPa and increases the \( \text{HVB} \) to about 1.5 times of that of the specimen. Therefore, a collimator with \( 2b_\psi = 0.5 \) mm was proposed.

**Collimator of \( 2b_\psi = 0.5 \) mm and \( C\ell = 30 \) mm**

Figure 8 shows the intrinsic stress and the \( \text{HVB} \) of profile at \( \psi_0 = 0 \) deg for various lengths of the collimators of \( 2b_\psi = 0.5 \) mm. It is seen that the collimator of 30 mm length has an intrinsic stress of -1 MPa and \( \text{HVB} \) similar to that of the specimen. Using Eq.(5), the stress error \( \Delta \sigma_L \) was calculated and plotted against \( L \) in Fig.9. It is seen in the figure that an error of about 15 MPa occurs for \( L = 0.1 \) mm.

The stress error due to collimator rotation for \( C\ell = 30 \) mm was calculated using Eq.(7) and plotted.
Table 1. Summary of the stress error for a goniometer of $R_0 = 30$ mm and collimator of $2b_\phi = 0.5$ mm and $C_\ell = 30$ mm.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Stress error (MPa)</th>
<th>Conditions assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic stress</td>
<td>~0 MPa</td>
<td></td>
</tr>
<tr>
<td>Due to specimen mis-setting</td>
<td>~15 MPa</td>
<td>$L=0.1$ mm</td>
</tr>
<tr>
<td>Due to collimator translation</td>
<td>~20 MPa</td>
<td>$t=0.05$ mm</td>
</tr>
<tr>
<td>Due to collimator rotation</td>
<td>~5 MPa</td>
<td>$\kappa=0.025$ deg</td>
</tr>
<tr>
<td>Due to PSD misalignment</td>
<td>~0 MPa</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>~40 MPa</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8. Intrinsic stress and half-value breadth against collimator length.

against $\kappa$ in Fig. 10. It can be seen in the figure that an error of about 5 MPa occurs for $\kappa = 0.025$ deg. The error due to collimator translation is independent of the size of collimator as shown in Eq. (6).

**Summary of Stress Error**

The stress errors under the conditions assumed by the authors are summarized in Table 1. The total error obtained is 40 MPa and seems not too large for the combination of a goniometer of $R_0 = 30$ mm and collimator of $2b_\phi = 0.5$ mm and $C_\ell = 30$ mm to be rejected, though it is necessary to further discuss whether the conditions assumed in Table 1 are conservative or not.

**SINGLE EXPOSURE METHOD**

The miniaturization of an analyzer using the single exposure method at $\psi_0 = 45$ deg was further
investigated. The simulation of X-ray stress measurement using this analyzer is easily conducted by revising the simulation program for the \( \Omega \) assembly X-ray stress analyzer. Conditions of simulation are same as those for the \( \Omega \) assembly analyzer except the incident angles.

**Intrinsic Stress**
Using a collimator of \( 2b_y = 0.5 \) mm and \( Cl = R_0 \), the relation between the intrinsic stress and the goniometer radius was calculated. It was found that the intrinsic stress increases with a decrease in the goniometer radius, but the values of stress at short goniometer radius are not high when compared with those of \( \Omega \) assembly analyzer.

**Misalignment of Detector**
Giving three modes of misalignment, translation \( t_d \) in the direction of diffraction angle, translation \( r_d \) in the direction of goniometer radius and rotation \( \kappa_d \) at detector center, to the detector at \( +\eta \) side, the effect of misalignment of the detector was investigated. The stress error \( \Delta \sigma_{td} \) due to \( t_d \) was large especially at short region of \( R_0 \). The stress errors of \( \Delta \sigma_{rd} \) and \( \Delta \sigma_{pd} \) due to \( r_d \) and \( \kappa_d \), respectively, showed the dependency on specimen stress but were found negligible even at the stress level of 500 MPa.

**Misalignment of Collimator**
The stress error \( \Delta \sigma_t \) due to collimator translation \( t \) increased with the decrease in the goniometer radius. The stress error \( \Delta \sigma_\kappa \) due to rotation of \( \kappa \) also increased with the decrease in the goniometer radius.

**Specimen Mis-setting**
The stress error due to specimen mis-setting was calculated for various collimator sizes, goniometer radius and specimen stress. It was found that the relation between the stress error and the offset depends on the various parameters, but that the value of error is extremely small. The stress error due to the inaccurate incident angle was also calculated and found negligibly small.

**Summary of Stress Error**
The total error with its components is given in Fig. 11. From the figure, it is seen that, even at \( R_0 = 200 \) mm, this method has a high value of total error. However, the main source of the error is from the assembly of components including X-ray tube, collimator and detectors. The experimental errors including specimen mis-setting and inaccurate incident angle of X-ray beam are small. Therefore, the single exposure method has the possibility of miniaturizing the size, if an advanced technique in components assembly could be adapted.

**REFERENCE**